

# A leapfrogging technique to materialize long traceable reference distances for EDM calibration

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**Abstract.** Periodic calibration of electronic distance meters is required to assess measurement errors and ensure metrological traceability. The high cost of reference measurement systems has been a barrier for the broader availability of calibration services. A method based on a short length standard and a leapfrogging technique to materialize traceable reference distances is proposed. A 50 m distance could be materialized with an expanded uncertainty of 0.0028 m using a simple prototype. Improving the mechanical design of the prototype may reduce uncertainty significantly.

## 1. Introduction

Electronic distance meters (EDM) are widely applied in civil engineering, topography as well as in the construction of large equipment such as ships, aircrafts and turbines for power plants. EDM are available as stand-alone systems or as length measurement devices in total stations. The strength of this technology lies in its capability of contact-free measuring distances of up to some hundreds of meters with high accuracy [1].

As with any measurement system, EDM have to be periodically calibrated to assess measurement errors and provide metrological traceability. Direct and indirect calibration methods may be considered. The direct method consists in measuring calibrated distances with the EDM. These distances are typically materialized by a baseline made up of a set of concrete pillars with some sort of coupling for reflectors. Laser trackers or high accuracy total stations are used to calibrate the distances between the pillars of the baseline [2]. On the indirect method, a given displacement is simultaneously measured with the EDM and a reference measurement system. This procedure is normally conducted on interferometric benches with laser interferometers as reference equipment [3,4]. The infrastructure for this type of calibration is sophisticated and generally restricted to National Metrology Laboratories.

The high cost of reference measurement systems is a barrier for the broader availability of accredited EDM calibration service providers. In this work we propose an alternative technique to materialize traceable reference distances for the direct calibration of EDM.

## 2. Leapfrogging method to materialize traceable reference lengths

In our method we revisit a leapfrogging technique that, in the past, was applied to generate baselines for geodetic networks [4]. Our goal is to evaluate the uncertainty that can be achieved with present levels of accuracy for precision manufacturing and measurement.

The basic concept of our method is illustrated in Figure 1. The idea is to use a short length standard and a leapfrogging technique to materialize a traceable distance between two pillars (A and B). The length standard (C) features two 3-DoF kinematic couplings (KC) for precision spheres (e.g. three contact points or conical nests). The distance between the centers of the two spheres placed on the KC's of the standard is calibrated with a high accuracy coordinate measuring machine (CMM). The measuring volume of the CMM restricts the maximum length of the standard.

Distance between pillars A and B is approximately a multiple of the length of the standard. The length of the standard is successively transferred to materialize the final distance between pillars A and B. Auxiliary devices (D) placed on tripods are used in the length transfer processes. The pillars as well as the length transfer devices feature KC's that are identical to the ones of the standard. KC of pillar A is fixed.

On the first length transfer, a tripod with a transfer device is placed in front of pillar A in a way that its KC is aligned to pillars A and B. Spheres are placed in the KC of both the pillar and the transfer device. The KC of the transfer device can be freely moved in the direction of the length to be materialized and fastened after the length transfer. During the length transfer, the KC's of the reference standard are settled on the spheres and the KC of the transfer device is fastened. In the next step, a second transfer device is placed in front of the first and the length of the bar is transferred as previously described.

This process is repeated until pillar B is reached after the  $n$ th length transfer. KC of pillar B is also initially movable and is fixed after the last length transfer. For an hypothetically error free process, distance between pillars A and B would be equal to  $n$  times the length of the standard.

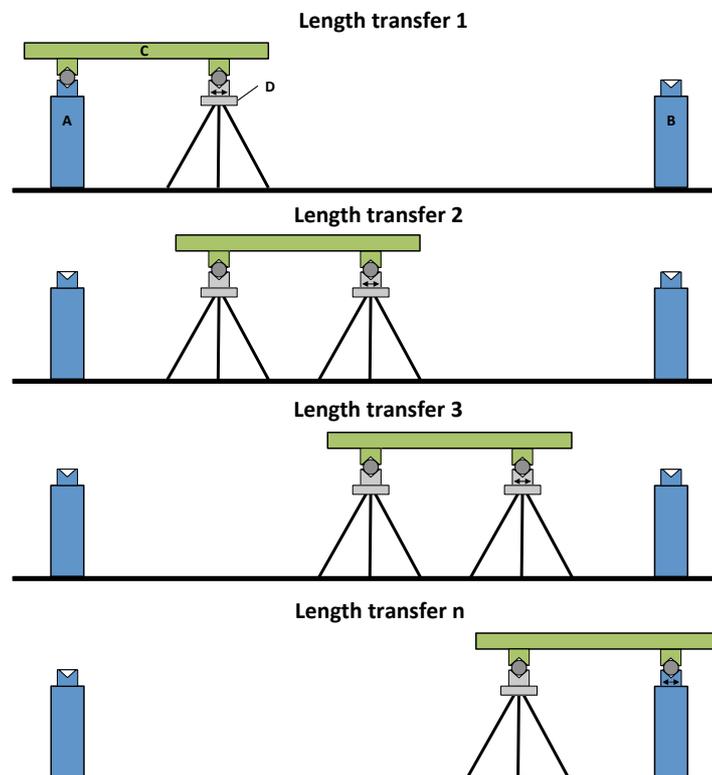


Figure 1: Leapfrogging technique.

### 2.1. Mathematical model of the measurement process and sources of uncertainty

The materialized distance after the  $i$ -th length transfer  $L_i$  is calculated according to Equation 1, with  $L_0 = 0$ .

$$L_i = L_{i-1} + L_S + L_S \alpha_S (T_i - 20^\circ\text{C}) - L_S (1 - \cos \theta) + e_{trans} + e_{stab} \quad (1)$$

Where,  $L_S$ : calibrated length of the length standard,  $\alpha_S$ : coefficient of linear thermal expansion of the length standard,  $T$ : temperature of the length standard during the  $i$ -th length transfer in  $^\circ\text{C}$ ,  $\theta$ : average misalignment angle,  $e_{trans}$ : error due to the length transfer process,  $e_{stab}$ : error due to the stability of the KC between length transfers.

The sources of uncertainty are summarized in Table 1.

**Table 1: Sources of uncertainty.**

Symbol	Description	Quantification	Distribution	Sensitivity coefficient
$U(L_S)$	Uncertainty of the length of the standard	Calibration certificate	Normal	$\alpha_S (T - 20^\circ\text{C}) + \cos \theta$
$U(\alpha_S)$	Uncertainty of the thermal expansion coefficient of the standard	Material data sheet	Rectangular	$L_S (T - 20^\circ\text{C})$
$U(T)$	Uncertainty of the temperature measurement	Calibration certificate	Normal	$L_S \alpha_S$
$U(\theta)$	Uncertainty due to misalignment	Experimental	Normal	$-L_S \sin \theta$
$e_{trans}$	Error due to of the length transfer process	Experimental	Normal	1
$e_{stab}$	Error due to the stability of the KC between transfers	Experimental	Normal	1

### 3. Test of concept

Figure 2 show the prototypes constructed to test the leapfrogging technique. An aluminum profile is used as length standard (Figure 2-a). The length standard features two conical nests for 38.1 mm spheres (detail in Figure 2-a). The center-to-center distance is approximately 1120 mm. The actual distance was calibrated on a CMM. The length transfer devices were also constructed from aluminum profiles and mounted on tripods (Figure 2-b). A t-nut is used to fasten the conical nests after the length transfer (Figure 2-c). A length transfer is shown in Figure 2-d. Figure 2-e illustrates the length standard being settled on the movable nest of the transfer device.

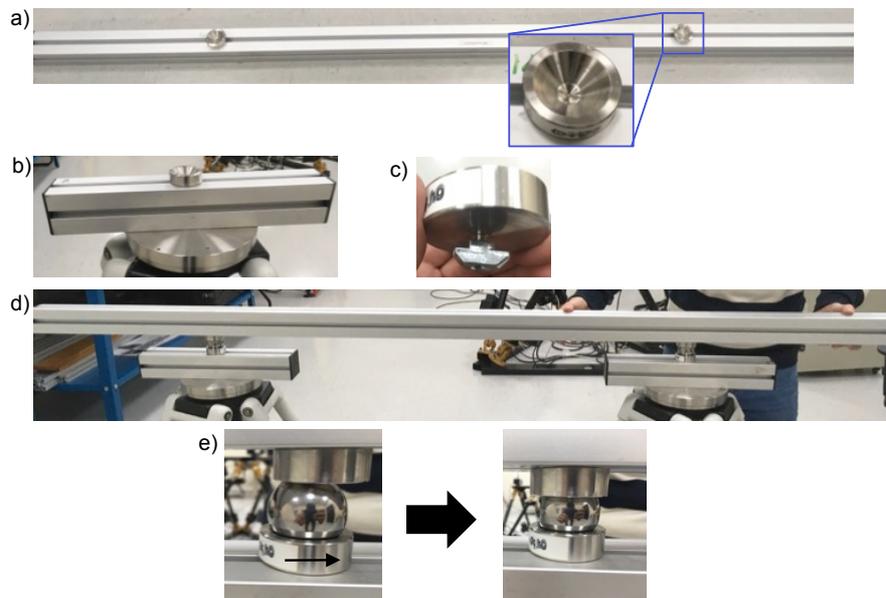


Figure 2. Prototype equipment.

We performed three independent experiments with 45 length transfers each, thus resulting in a final distance of approximately 50.5 m. The experiments were conducted in a long corridor. Tripods emulated the pillars. The setup is shown in Figure 3-a and b. The length generated after the  $i$ -th length transfer was calculated according to Equation 1, i.e., results were corrected for thermal expansion (temperature measured for each transfer) and average misalignment angle. Each experiment was completed after approximately four hours.

A robotic total station (TS) was used to measure the position of the conical nests of both transfer devices after each length transfer. The laser beam of the TS was also used to align the transfer devices. A 38.1 mm diameter spherical mounted reflector (SMR) served as target (Figure 3-c).

Our total station is a Leica TS 12, manufactured in 2015. The precision (repeatability) of its EDM, according to ISO 17123-4, is  $1.0 \text{ mm} + 1.5 \text{ ppm}$ . This specification was used as estimate for the uncertainty of the TS results. Resolution is 0.1 mm.

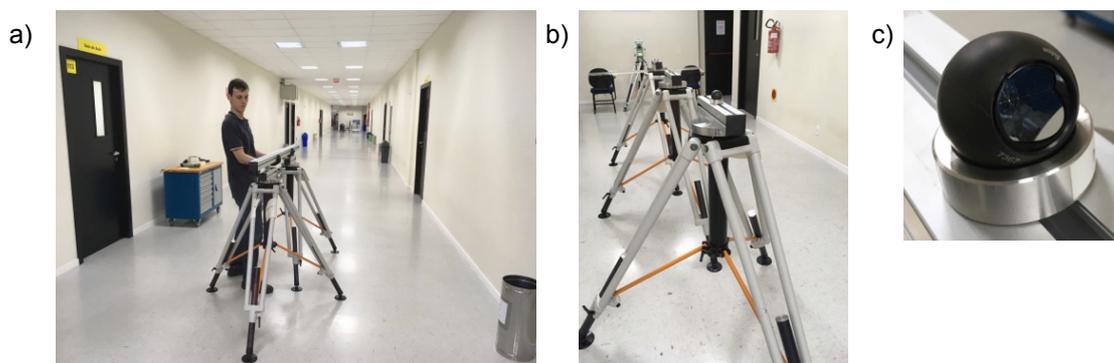


Figure 3: Experiment setup.

The measurement uncertainty budget for a length transfer process is show in Table 2. Repeatability of the length transfer mechanism was estimated through an experiment on a CMM. Average misalignment and respective uncertainty were quantified with the TS data collected during the experiment on the corridor. Stability of the position of the kinematic nests between two consecutive

transfers was also estimated with the experimental data. This latter source of uncertainty is clearly dominant.

**Table 2: Standard uncertainty of a length transfer.**

	Mean	Uncertainty	Divisor	Sensitivity coefficient	u [mm]	Degrees of freedom	% of contribution
$L_S$	1121.6056 mm	0.0038 mm	2	0.9997	0.0019	$\infty$	0.01
$\alpha_S$	0.000023 1/°C	0.000001 1/°C	$\sqrt{3}$	6955	0.0040	$\infty$	0.04
T	26.2°C	0.5°C	2	0.026	0.0065	$\infty$	0.10
$\theta$	0.0038 rad	0.0021 rad	1	4.31	0.009	134	0.18
$e_{trans}$	0	0.0126 mm	1	1	0.0126	4	0.36
$e_{stab}$	0	0.20 mm	1	1	0.21	134	99.32
<b><math>u_{c,Tf}</math></b>					<b>0.2065</b>	<b>135</b>	

Expanded uncertainty (k=2) of a length generated after the  $i$ th transfer is calculated according to Equation 2.

$$U = 2 \cdot \sqrt{i \cdot u_{c,Tf}^2} \quad (2)$$

The results of the experiments are plotted in Figure 4. Each point represents the difference between the distance generated by the leapfrogging method, calculated according to Equation 1, and the distance indicated by the TS after each length transfer. Dashed lines stand for the expanded uncertainty calculated according to Equation 2. Error bars are proportional to the uncertainty of the TS measurements, estimated by the precision specified by the manufacturer. Figure 4 also compares the results of the length materialized with our method with the length measured by the TS after the last length transfer as well as the average temperature in each experiment. The smaller final length of the third experiment compared to the other two is clearly related to the lower average temperature during this experiment.

The fact that the regions of uncertainty of the leapfrogging method and the TS measurements overlap for all points indicate that results are likely to be consistent. It is also important to note, that the uncertainty of the TS measurements may be underestimated, since the specification for precision only accounts for random errors.

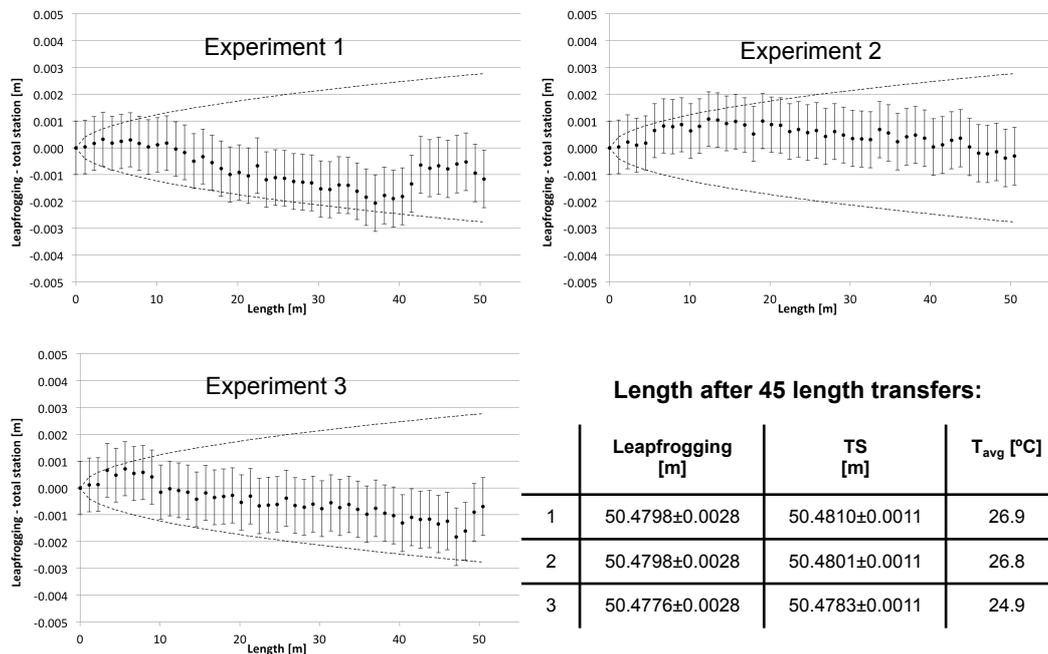


Figure 4: Results of the test of concept.

#### 4. Discussion and conclusions

Although within limits of uncertainty, there is an apparent negative drift in the results of all three experiments. The reason for this effect is not clear. One of the possible causes is that the average misalignment error is actually smaller than the value that we used to correct the results (0.0038 rad). In this case, we would have “overcorrected” the data. Additional tests have to be carried out to confirm this hypothesis.

The dominant source of uncertainty for the test of concept is the stability of the KC between two length transfers. Apparently, small displacements of the fixed KC are occurring during the length transfer process. Improving the mechanical design of the devices may reduce this error. A 50% reduction of this source of uncertainty would make the final expanded uncertainty for a 50 m length to reduce to 1.5 mm. The uncertainty would also benefit from a longer length standard, since the final uncertainty is proportional to the number of length transfers. The redesign of the equipment should also aim at reducing the errors related to misalignment, reducing the time to complete the procedure and increasing robustness to temperature variations.

It is important to mention, that we only evaluated the uncertainty of the leapfrogging method. Additional sources of uncertainty should be taken into account on using the materialized distance for EDM calibration, such as the long-term stability of the distance and errors that result from adapting appropriate targets to the KC of the pillars.

With this research we demonstrated the potential of the leapfrogging method to materialize long traceable lengths. Efforts will be concentrated at designing and constructing new prototypes to reduce uncertainty of the materialized lengths.

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