

Experimental-numerical approach to the estimation of uncertainty in robot pose measurement

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Abstract. Accurate pose measurements are crucial for the reliability of the kinematic calibration and performance evaluation of a robotic manipulator. Spherical coordinate measurement systems such as laser trackers and total stations are commonly used to collect sets of 3D points from which position and orientation of a moving coordinate frame attached to the end-effector in relation to a reference coordinate frame are calculated. Estimating the uncertainty of this specific measurement task is necessary to provide traceability to these measurements. In this paper we propose combining measurements of calibrated artefacts with Monte Carlo simulations to estimate this task-specific measurement uncertainty. The application of this method has been demonstrated for the performance evaluation of a Stewart Platform using a robotic total station. Expanded uncertainties between 0.40 mm and 0.63 mm have been estimated for position. Expanded uncertainty was 0.10° for all orientation results.

Keywords: robot calibration, pose accuracy, total station, laser tracker, measurement uncertainty

1. Introduction

The pose of a robotic manipulator is the position and orientation of its end-effector after a given displacement. During kinematic calibration or performance evaluation processes several poses are generated with the robot and the actual position and orientation of its end-effector assessed with appropriate measurement equipment. Spherical coordinate measurement systems such as laser trackers and high accuracy total stations are used to collect 3D points from which the pose is calculated. In order to provide metrological traceability it is necessary to state the uncertainty of this specific measurement task.

Due to the complexity of measurements with coordinate measurement systems, the task-specific uncertainty estimation is typically addressed by numerical or experimental methods. The German National Metrology Laboratory PTB proposed in the 1990's a solution for the numerical estimation of task-specific measurements with cartesian coordinate measurement machines known as "Virtual-CMM" which is available in commercial software [1]. A version for laser trackers is currently under development in the United Kingdom [2]. A limitation of this method is that several sources of uncertainty related to factors such as equipment geometry, probing and thermal expansion have to be quantified through a sophisticated calibration. An alternative is the experimental uncertainty evaluation described in ISO 15530-3 [3]. This method is based on repeated measurements of a calibrated workpiece. The calibrated workpiece and the condition during these reference measurements have to be sufficiently similar to the measurement process under evaluation. This requirement is not easily fulfilled for the specific measurement problem discussed in this paper.

We therefore propose a strategy that combines measurements of simple calibrated artefacts with Monte Carlo simulations. This method is limited to applications in which the pose is assessed through a set of 3D point measurements.

2. Experimental-numerical method

The proposed procedure to estimate the uncertainty of pose measurements is illustrated in the flow chart in Figure 1. The first step is to define a mathematical model that relates the pose of the end effector to sets of 3D points measured to define the reference base frame b and the moving frame p . The remaining steps will be outlined in Sections 2.1, 2.2 and 2.3.

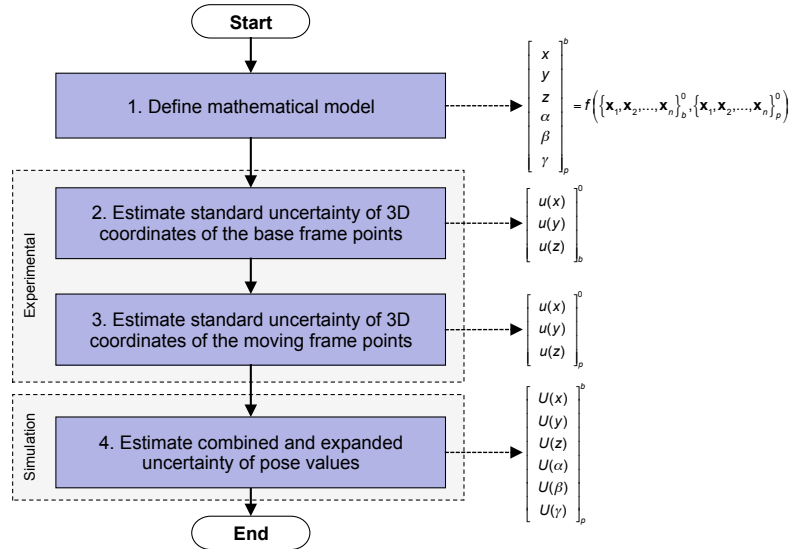


Figure 1. Uncertainty estimation procedure.

2.1. Standard uncertainty of the coordinates of the base frame points

In this step the standard uncertainties of the coordinates of the points that define the stationary base frame b are estimated experimentally. We propose to evaluate these uncertainties through a set of measurements of a calibrated 2D artefact. If a spherical mounted reflector (SMR) is used, the reference points of this artefact would typically be materialized through kinematic nests. It is necessary for the artifact to have some more points than the number of points used to define frame b and that the positions of the points are similar to the distribution of the equivalent points on the robot.

The measurement end evaluation sequence is the following:

- 1) Place 2D artefact in a position and orientation in relation to the measurement system comparable to the position and orientation of the points that define frame b in relation to the measurement system during the actual measurement on the robot;
- 2) Measure the points that define the coordinate frame of the artefact and calculate frame;
- 3) Measure coordinates of all m points of the artefact n times in respect to the artefact frame. We propose a minimum of $n = 3$ measurements;
- 4) Calculate measurement errors of the coordinates for all $j = \{1 \dots m \ n\}$ points according to Equation 1. If temperature of the artefact is monitored, actual values may be corrected to 20°C;

$$\begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}_j = \begin{bmatrix} x_{act} - x_{cal} \\ y_{act} - y_{cal} \\ z_{act} - z_{cal} \end{bmatrix} \quad j = \{1 \dots m \ n\} \quad (1)$$

- 5) Calculate mean and standard deviation values of e_x , e_y and e_z ;
- 6) Evaluate standard uncertainties u_x , u_y and u_z according to Equation 2. We opted to treat the uncorrected bias as a standard uncertainty, as recommended in [4].

$$u_i = \sqrt{u_{cal}^2 + u_{var,i}^2 + u_{T,i}^2 + b^2} \quad i = \{x, y, z\} \quad (2)$$

where, u_i : standard uncertainty of coordinate i ; u_{cal} : maximum standard uncertainty of the calibrated coordinate values of the standard artefact; $u_{var,i}$: variability of coordinate i quantified as the standard deviation of the measurement errors of coordinates i ; $u_{T,i}$: standard uncertainty due to the thermal expansion of the 2D artefact; b_i : bias (average value of e_i).

The standard uncertainty due to thermal expansion is calculated according to Equation 3. A uniform probability distribution is assumed.

$$u_{T,i} = \frac{L_{max} \alpha \Delta T}{\sqrt{3}} \quad i = \{x, y, z\} \quad (3)$$

where, L_{max} : maximum value of coordinate i ; α : thermal expansion coefficient of the material of the 2D artefact; ΔT : maximum absolute difference between temperature of the artefact and 20°C or expanded uncertainty of the temperature measurement if temperature is corrected.

2.2. Standard uncertainty of the coordinates of the moving frame points

The uncertainties of the coordinates of the points that define moving frame p are estimated experimentally through a set of measurements of a calibrated length standard. Again, the points that define the length of the artefact would typically be materialized through kinematic nests. The standard should have a length that is similar to the largest distance between the points that define both frames of the robot considering all measured poses. Also, the positions of the bar in relation to the measurement system during the experiment should to be comparable to the condition during the measurements on the robot.

The length standard is measured in m different positions/orientations. These positions/orientations have to be carefully chosen in order to assure that the measurements are sensible to the main geometrical error components of the measurement system that would affect the actual measurements on the robot. We propose the measurement of at least six 1D positions (two in each axis x , y , z), six 2D diagonals (two in each plane xy , yz , zx) and two 3D diagonals, i.e. at least 14 positions/orientations in total, as depicted in Figure 2.

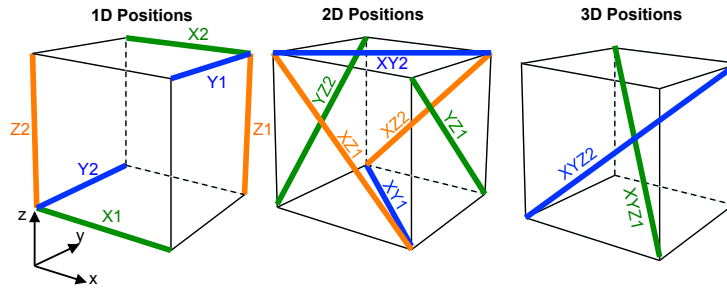


Figure 2: Positions for length standard measurements.

The measurement end evaluation sequence is the following:

- 1) Measure the length artefact in all m positions n times. We propose a minimum of $n = 3$ repeated measurements;
- 2) Calculate length measurement errors according to Equation 4. If temperature of the artefact is monitored, the actual values should be corrected to 20°C;
- 3) Calculate mean and standard deviation values of e_L ;
- 4) Evaluate standard uncertainties u_x , u_y and u_z according to Equation 2.

$$e_{L,j} = L_{act,j} - L_{cal} \quad j = \{1 \dots n \ m\} \quad (4)$$

$$u_i = \sqrt{u_{cal}^2 + u_{var}^2 + u_T^2 + b^2} \quad i = \{x, y, z\} \quad (5)$$

where, u_i : standard uncertainty of coordinate i ; u_{cal} : maximum standard uncertainty of the calibrated length of the standard; u_{var} : variability measured as the standard deviation of the measurement errors; u_T : standard uncertainty due to the thermal expansion of the length standard; b_i : bias (average of $e_{L,j}$ values).

The standard uncertainty due to thermal expansion is calculated according to Equation 6. A uniform probability distribution is assumed.

$$u_T = \frac{L \alpha \Delta T}{\sqrt{3}} \quad (6)$$

where, L : length of the standard; α : thermal expansion coefficient of the material of the length standard; ΔT : maximum absolute difference between temperature of the artefact and 20°C or expanded uncertainty of the temperature measurement if temperature is corrected.

2.3. Combined and expanded uncertainty of pose values

Standard uncertainties of the coordinates of the points that define frames b and p are used as standard deviations of normal probability distributions to be propagated in the Monte Carlo simulations. If the uncertainty estimation is performed *a priori*, nominal coordinate values of the robot measurements can be used as mean values of the distributions. If it is an *a posteriori* evaluation, mean values should be the average indications.

A sufficiently large number M of Monte Carlo trials should be performed. We use $M=1e6$, as recommended by the Supplement 1 to the GUM [4]. Values for the expanded uncertainties should only be stated if assumption of normal distribution of the output values is acceptable.

3. Uncertainty of pose measurements for the performance evaluation of a Stewart Platform

The measurement problem to be discussed in this section is the determination of pose values of a Stewart Platform with a robotic total station. The purpose of these measurements is the evaluation of the performance of the equipment. Our Stewart Platform is depicted in Figure 3-a. A Stewart Platform is a 6-DoF robotic manipulator. The pose of the upper moving platform in relation to the stationary base is changed by appropriately varying the lengths of six legs. The legs of our equipment are lead screw actuators driven by servos. The legs are attached to the base and to the moving platform by means of universal joints.

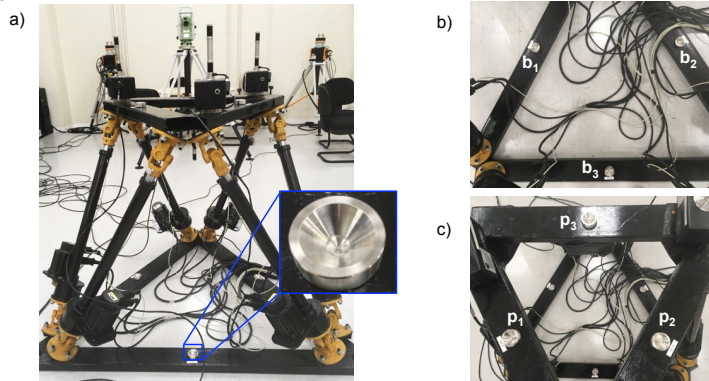


Figure 3: Stewart Platform.

Frames b and p are defined by sets of three conical nests that are designed for 38.1 mm diameter SMR (Figure 3-b and c). The three points that define the base and the platform are denoted b_i and p_i , respectively.

Our total station is a Leica TS 12, manufactured in 2015. The precision (repeatability), according to ISO 17123-4, is 1.0 mm + 1.5 ppm for length measurement and 7" for angle measurement. Resolution is 0.1 mm.

3.1. Mathematical model

Coordinate frames b and p are defined by the classic 3-2-1 method, i.e., points 1, 3 and 3 define the primary reference (the xy-plane), points 1 and 2 points define the secondary reference (the x-axis) and point 1 define the origin. Mathematically this definition can be described as follows.

Let b_i and p_i be 3D coordinates in mm described in an arbitrary frame "0".

$$\mathbf{b}_i^0 = [x, y, z]^T \quad i = \{1, 2, 3\} \quad (7)$$

$$\mathbf{p}_i^0 = [x, y, z]^T \quad i = \{1, 2, 3\} \quad (8)$$

The unit vectors and the rotation matrix of frame b are:

$$\mathbf{x}_b^0 = [x_{b,x}, x_{b,y}, x_{b,z}]^T = \frac{\mathbf{b}_2^0 - \mathbf{b}_1^0}{\|\mathbf{b}_2^0 - \mathbf{b}_1^0\|} \quad (9)$$

$$\mathbf{z}_b^0 = [z_{b,x}, z_{b,y}, z_{b,z}]^T = \frac{(\mathbf{b}_3^0 - \mathbf{b}_1^0) \times \mathbf{x}_b^0}{\|(\mathbf{b}_3^0 - \mathbf{b}_1^0) \times \mathbf{x}_b^0\|} \quad (10)$$

$$\mathbf{y}_b^0 = [y_{b,x}, y_{b,y}, y_{b,z}]^T = \mathbf{z}_b^0 \times \mathbf{x}_b^0 \quad (11)$$

$$\mathbf{R}_b^0 = \begin{bmatrix} x_{b,x} & y_{b,x} & z_{b,x} \\ x_{b,y} & y_{b,y} & z_{b,y} \\ x_{b,z} & y_{b,z} & z_{b,z} \end{bmatrix} \quad (12)$$

The unit vectors and the rotation matrix of frame p are:

$$\mathbf{x}_p^0 = [x_{p,x}, x_{p,y}, x_{p,z}]^T = \frac{\mathbf{p}_2^0 - \mathbf{p}_1^0}{\|\mathbf{p}_2^0 - \mathbf{p}_1^0\|} \quad (13)$$

$$\mathbf{z}_p^0 = [z_{p,x}, z_{p,y}, z_{p,z}]^T = \frac{\mathbf{x}_p^0 \times (\mathbf{p}_3^0 - \mathbf{p}_1^0)}{\|\mathbf{x}_p^0 \times (\mathbf{p}_3^0 - \mathbf{p}_1^0)\|} \quad (14)$$

$$\mathbf{y}_p^0 = [y_{p,x}, y_{p,y}, y_{p,z}]^T = \mathbf{z}_p^0 \times \mathbf{x}_p^0 \quad (15)$$

$$\mathbf{R}_p^0 = \begin{bmatrix} x_{p,x} & y_{p,x} & z_{p,x} \\ x_{p,y} & y_{p,y} & z_{p,y} \\ x_{p,z} & y_{p,z} & z_{p,z} \end{bmatrix} \quad (16)$$

Frame origins are shifted from point 1 to locate them approximately at the center of the base and the moving platform:

$$\mathbf{o}_b^0 = \mathbf{b}_1 + \mathbf{R}_b^0[344, -196, 0]^T \quad (17)$$

$$\mathbf{o}_p^0 = \mathbf{p}_1 + \mathbf{R}_p^0[196, 111, 0]^T \quad (18)$$

A given platform pose is described through vector \mathbf{o}_p^b (origin of the platform frame in respect to the base frame) and rotation matrix \mathbf{R}_p^b :

$$\mathbf{o}_p^b = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_p^b = \mathbf{o}_p^0 - \mathbf{o}_b^0 \quad (19)$$

$$\mathbf{R}_p^b = \mathbf{R}_p^0(\mathbf{R}_b^0)^{-1} = \mathbf{R}_p^0(\mathbf{R}_b^0)^T = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \quad (20)$$

In this work we adopt the ZYX (roll, pitch, yaw) angle convention. Let α , β and γ be the rotations of the platform frame in respect to the base frame about axes x , y and z , respectively. These angles are calculated from the rotation matrix using the $Atan2$ function:

$$\alpha = Atan2(r_{23}, r_{13}) \quad (21)$$

$$\beta = Atan2(\sqrt{r_{13}^2 + r_{23}^2}, r_{33}) \quad (22)$$

$$\gamma = Atan2(r_{32}, -r_{31}) \quad (23)$$

3.2. Experimental estimation of the standard uncertainty of the 3D coordinates of points \mathbf{b}_i

We constructed a simple 2D artefact from aluminum profiles (Figure 4-left). The artefact features six conical nests for 38.1 mm SMR as probing elements and was calibrated on a coordinate measuring machine. Points 1, 2 and 3 define the xy -plane, x -axis points from point 1 to point 2 and point 1 defines the origin. Nominal distance between points 1 and 2 is 430 mm and nominal distance between points 1 and 4 is 605 mm.

During the measurements, position and orientation of the total station in relation to the artefact (Figure 4-right) was comparable to the pose measurements on the Stewart Platform. The position of each nest was measured three times. Temperature varied between 19.6°C and 19.7°C. The expanded uncertainty of the temperature measurement was estimated to be $U = 0.3^\circ\text{C}$. We thus considered a maximum deviation from 20°C of $\Delta T = 0.7^\circ\text{C}$. Thermal expansion coefficient is $23 \times 10^{-6} \text{ 1/K}$.

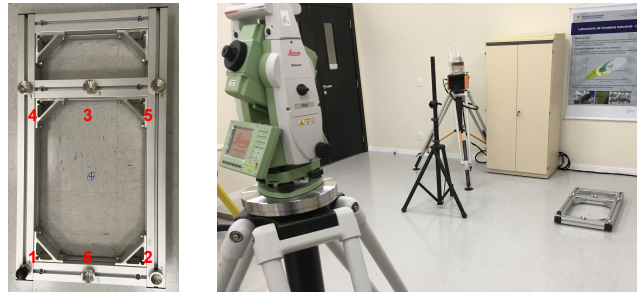


Figure 4: 2D artefact.

Figure 5 presents the coordinate measurement errors calculated according to Equation 1. Coordinates of point 1, as well as y and z coordinates of point 2 and z coordinate of point 3 were not included in the analysis, since these coordinates define the reference frame of the artefact. The dashed blue lines stand for the average errors (bias).

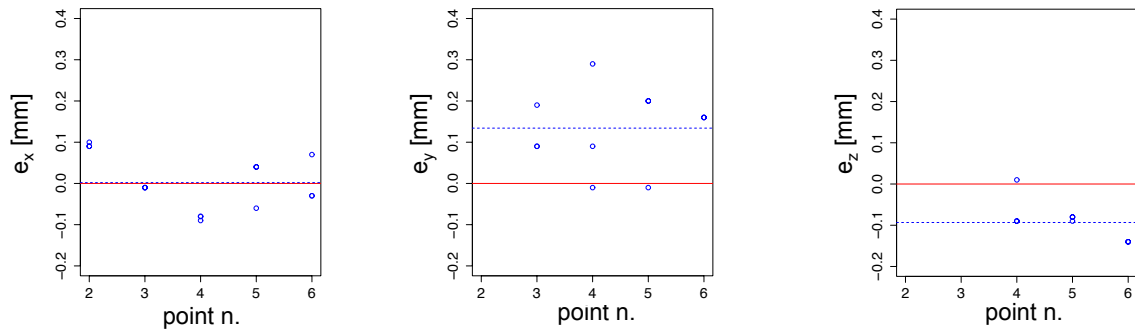


Figure 5: Coordinate measurement errors assessed with the 2D artefact.

The values of the uncertainty sources and the combined uncertainties for the coordinates of points b_i are listed in Table 1.

Table 1: Uncertainty sources and combined uncertainty of coordinates of points b_i .

Coordinate	u_{cal} [mm]	u_T [mm]	u_{var} [mm]	b [mm]	u_c [mm]
x	0.005	0.006	0.066	0.002	0.066
y			0.088	0.134	0.110
z			0.047	-0.093	0.105

3.3. Experimental estimation of the standard uncertainty of the 3D coordinates of points p_i

The length standard was also constructed from an aluminum profile. The reference length of 1120 mm is defined by two conical nests and was calibrated using a CMM as well.

The length standard was measured three times in each position shown in Figure 2. Measurement of positions “Z1” and “XYZ2” are depicted in Figure 6-a and b, respectively. Temperature varied between 19.8°C e 20.0°C. By taking the uncertainty of the temperature values into account, $\Delta T = 0.5^\circ\text{C}$ was adopted.

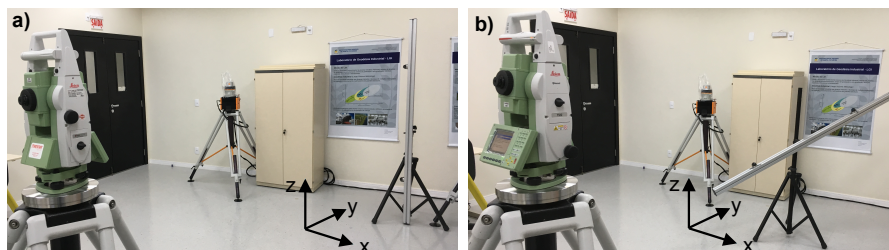


Figure 6. Measurement of the length standard.

Length measurement errors calculated according to Equation 4 are plotted in Figure 7 and values of the uncertainty sources and the combined uncertainty presented in Table 1.

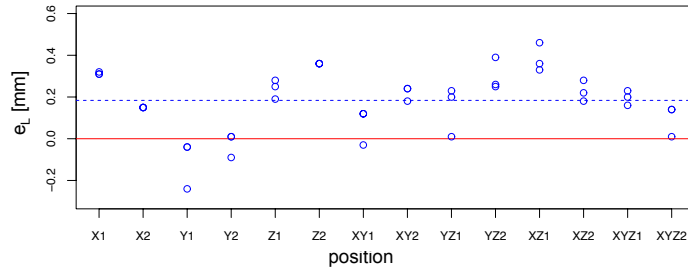


Figure 7: Length measurement errors.

Table 2: Uncertainty sources and combined uncertainty of coordinates of points p_i .

Coordinate	u_{cal} [mm]	u_T [mm]	u_{var} [mm]	b [mm]	u_c [mm]
x, y, z	0.005	0.008	0.148	0.184	0.236

3.4. Simulation-based estimation of the expanded uncertainty of the pose measurement

Within the scope of the performance evaluation, each one of the 13 poses defined in Table 3 was controlled. The expanded uncertainty was estimated for each measurement result by means of Monte Carlo Simulations. The mathematical model and the Monte Carlo algorithm were implemented in *R* programming language.

Table 3: Nominal poses for the performance evaluation of the Stewart Platform.

	Pose nr.												
	1	2	3	4	5	6	7	8	9	10	11	12	13
x [mm]	-120	-120	0	120	120	0	0	0	0	0	0	0	0
y [mm]	120	-120	0	120	-120	0	0	0	0	0	0	0	0
z [mm]	1235	1235	1175	1115	1115	0	0	0	0	0	0	0	0
α [°]	0	0	0	0	0	-5	5	0	0	0	0	-5	5
β [°]	0	0	0	0	0	0	0	-5	5	0	0	-5	5
γ [°]	0	0	0	0	0	0	0	0	0	-5	5	-5	5

As an example, Figure 8 illustrates the frequency distributions of the simulation outputs for coordinate x and angle α of pose 13. The solid red lines are the mean values and the dashed blue lines the limits of uncertainty ($k = 2$). The normality of the output values was confirmed by means of a Shapiro-Wilk test. The distribution for the other coordinates and angles and the remaining poses is similar.

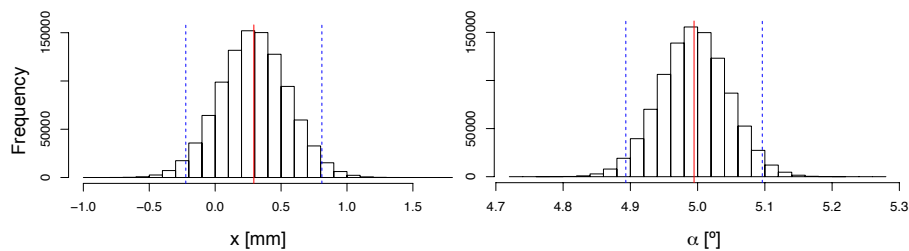


Figure 8. Frequency distribution of coordinate x and angle α .

The expanded uncertainties ($k = 2$) for the 13 poses are listed in Table 4.

Table 4: Expanded uncertainties for the 13 pose measurements.

	Pose nr.												
	1	2	3	4	5	6	7	8	9	10	11	12	13
$U(x)$ [mm]	0.52	0.53	0.51	0.48	0.50	0.51	0.51	0.51	0.51	0.50	0.52	0.50	0.52
$U(y)$ [mm]	0.63	0.63	0.61	0.60	0.60	0.61	0.61	0.61	0.61	0.62	0.60	0.62	0.60
$U(z)$ [mm]	0.40	0.40	0.40	0.41	0.41	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40
$U(\alpha)$ [°]	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
$U(\beta)$ [°]	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
$U(\gamma)$ [°]	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10

4. Discussion and conclusions

The method proposed in this paper is limited to circumstances in which pose is calculated from 3D points. Nonetheless, it is not limited to a specific coordinate measurement system or to a specific robotic manipulator. However, the design of the artefacts should always take the requirement of similarity to the actual measurement process into account. Also, the uncertainties related to the calibration of the artefact should be reduced if more accurate systems such as laser trackers are under evaluation. This can be done, for instance, by constructing artefacts from a material with lower thermal expansion coefficient.

It is also possible to use the results of the length measurements to estimate the uncertainty of the coordinates of frame b , as an alternative to using the 2D-artifact. However, in most cases, this will lead to an overestimation of the uncertainty. It is also possible to completely eliminate the experimental steps by using the specification for the maximum permissible 3D length measurement error or 3D point uncertainty as informed by the manufacturer of the measurement system. Nonetheless, this will probably lead to an even larger overestimation.

As mentioned in Section 2.1, we opted to treat the uncorrected bias as a standard uncertainty, against the GUM recommendation to always correct systematic errors. In our case, it is not possible to use the bias quantified in the experiments to correct individual pose values, since there is no functional relationship between both. The estimated bias should therefore be interpreted as an estimate for the average systematic coordinate measurement error.

The values for estimated uncertainty seem plausible. Nonetheless it was not yet possible for us to experimentally validate these values due to the lack of an appropriate reference measurement system. Efforts are being undertaken so that we have access to a laser tracker in the near future.

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