

# Sequential multilateration technique for the determination of 3D coordinates of reference points in industrial geodesy using a high-accuracy self-tracking laser interferometer

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*Multi-station measurements with instruments such as total stations and laser trackers often demand the use of stationary reference points (SRP) that define a global coordinate system. The accuracy in determining the position of these points has direct impact on the final measurement uncertainty. This paper presents a technique and a software tool developed for measuring the 3D coordinates of SRP by sequential multilateration with a self-tracking laser interferometer. The software tool is used for the multilateration calculations, for the estimation of measurement uncertainty by Monte Carlo simulation (MCS) as well as for the optimization of the positions of the measurement stations. The technique was applied to determine the positions of SRP of an indoor-GPS system. Experimental results are consistent with the simulations. For the target point with highest uncertainty the MCS results are for coordinates x, y and z resulted respectively 0,20 mm, 0,16 mm and 3,70 mm (shortest coverage interval for 95% coverage probability). The nature of the measurement task implies a worse sensitivity of the technique for the z-coordinate. The dominant source of uncertainty was identified to be the poor repeatability of the length measurements, which is basically determined by the accuracy of the hole-shaft fit of the adapter used to position the reflector on the target points. Means of reducing this influence are being investigated. Simulation indicates that it is possible to achieve experimental uncertainties of 30  $\mu$ m within a 5 m radius spherical range.*

## NOMENCLATURE

MCS = Monte Carlo Simulation

SRP = Stationary reference points

Monuments = stationary reference points of the iGPS system

u = standard uncertainty

## 1. Introduction

Increasing requirements for manufacturing and assembling processes of large parts and structures, mainly in the fields of aircraft and shipbuilding industries, are pushing forward the development and improvement of measurement technologies, especially those designed for flexible tridimensional metrology with low measuring uncertainty. This field of metrology is known as large-scale metrology or industrial geodesy. In industrial geodesy a global coordinate system defined by a set of stationary reference points is frequently used to reduce the uncertainty of multi-station measurements. The positions of these points have to be measured using a reference system with low measurement uncertainty. It is expected from a laser

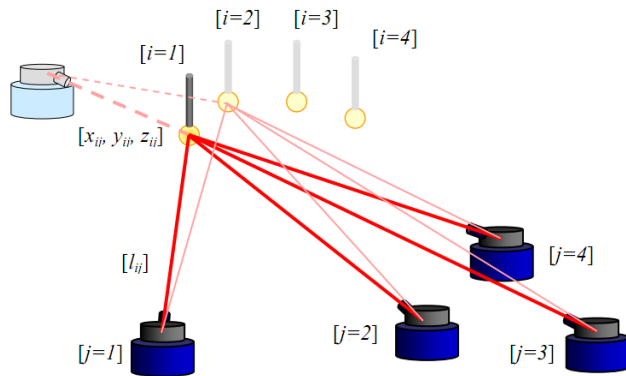
interferometer-based multilateration system to perform the measurement of 3D coordinates with high accuracy and direct traceability to the definition of meter.

This work presents a sequential multilateration technique using a self-tracking laser interferometer, commercially known as LaserTracer®, for measuring 3D coordinates of stationary reference points (SRP). This technique was applied for the measurement of SRP (monuments) of an indoor-GPS system. A software tool developed in MATLAB® and LabView® is used for the multilateration calculations, estimation of measurement uncertainty by Monte Carlo simulation (MCS) as well as for the search for the best configuration of measurement stations.

## 2. Sequential Multilateration using a LaserTracer®

Multilateration consists of determining 3D coordinates of a set of points from multiple distance measurements between these points and the measuring system. Figure 1 illustrates the measuring principle. At least four positions of the measuring system are needed. In sequential multilateration, length measurements to each target point (index *i*) are sequentially taken from each position of the measurement system

(each position is called measurement station; index  $j$ ). By using the measured distances as input, an overdetermined system of non-linear equations is solved in order to define the positions of the target points and of the measurement stations.



**Figure 1: Multilateration principle**

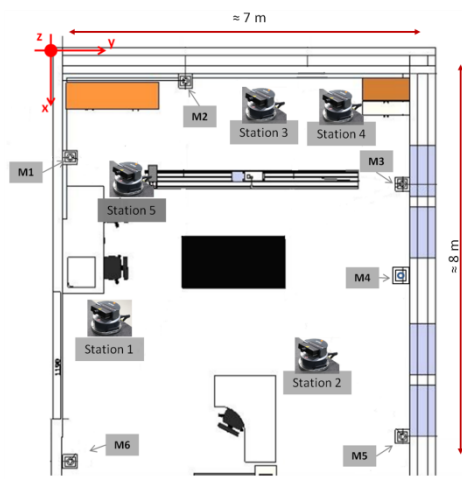
The LaserTracer® is a self-tracking laser interferometer optimized for length measurements. It has a precision sphere in its rotating center that acts as the reference for the length measurements, which become independent of the mechanical precision of the rotating mechanism. This allows achieving spatial length measurements with uncertainties about 10 times lower than using a Laser Tracker. This feature makes this system specially suited for a multilateration technique.

A software tool developed in MATLAB® and LabView® is used for: i) Multilateration calculations; ii) Search for the best positions of the measurement stations; iii) Estimation of measurement uncertainty by Monte Carlo simulation.

### 3. Case Study

This case study refers to the measurement of reference points (monuments) of an indoor-GPS system. This task was carried out in three main steps: 1) Finding the best positions of the measurement stations, given the approximate location of the monuments; 2) Estimating the measurement uncertainty related to this configuration of stations; 3) Carrying out the distance measurements, using the information gained with the previous simulations and calculating the coordinates of the reference points.

Figure 2 shows a scheme of the measurement layout in two dimensions. Legends M1 until M6 indicate the positions of the monuments. The positions of the stations (station 1 until 5) are those



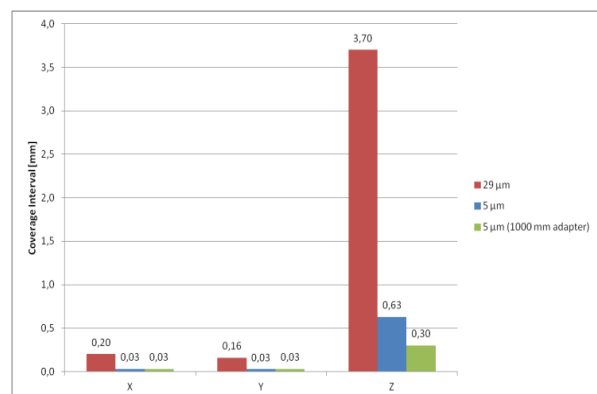
**Figure 2: Measurement layout**

obtained by simulation.

The measurement uncertainty resulting from the MCS are, for coordinates  $x$ ,  $y$  and  $z$ , respectively: 0,20 mm; 0,16 mm and 3,70 mm (shortest coverage interval for a coverage probability of 95,45%).

### 4. Discussion of the Results

The dominant source of uncertainty was identified to be the poor repeatability of the length measurements ( $u = 29 \mu\text{m}$ ), which is basically determined by the accuracy of a hole-shaft fit of the adapter used to position the reflector on the target points. Another simulation has been carried out in order to analyze this behavior, in which the uncertainty of length measurements was reduced to  $u = 5 \mu\text{m}$ . In fact, it implied a reduction of more than 5 times in the coverage intervals for all coordinates. It is also clear that the technique has a worse sensitivity in the  $z$ -coordinate. This comes from the nature of the measurement task, where the length variations in “ $z$ ” direction are much smaller than in the other directions. A third simulation considered an adapter that creates larger variations in “ $z$ ”, which reduced the uncertainty in this direction to half the previous value. Figure 3 shows the results of these simulations.



**Figure 3: MCS results**

### 5. Conclusions

A technique has been developed which makes use of a LaserTracer® and sequential multilateration for measuring 3D coordinates of stationary reference points used in industrial geodesy. Simulations indicate that it is possible to overcome experimental limitations, achieving high accuracy coordinate measurements with direct traceability to the definition of meter.

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